RBE 3005
วิศวกรรมหุ่นยนต์ (Robotics Engineering)

สาขาวิศวกรรมหุ่นยนต์

คณะวิศวกรรมศาสตร์และเทคโนโลยีอุตสาหกรรม

มหาวิทยาลัยราชภัฏสวนสุนันทา

จลนศาสตร์ผกผัน (Inverse Kinematics)

2-Dimensional (Planar) Robotic Arms

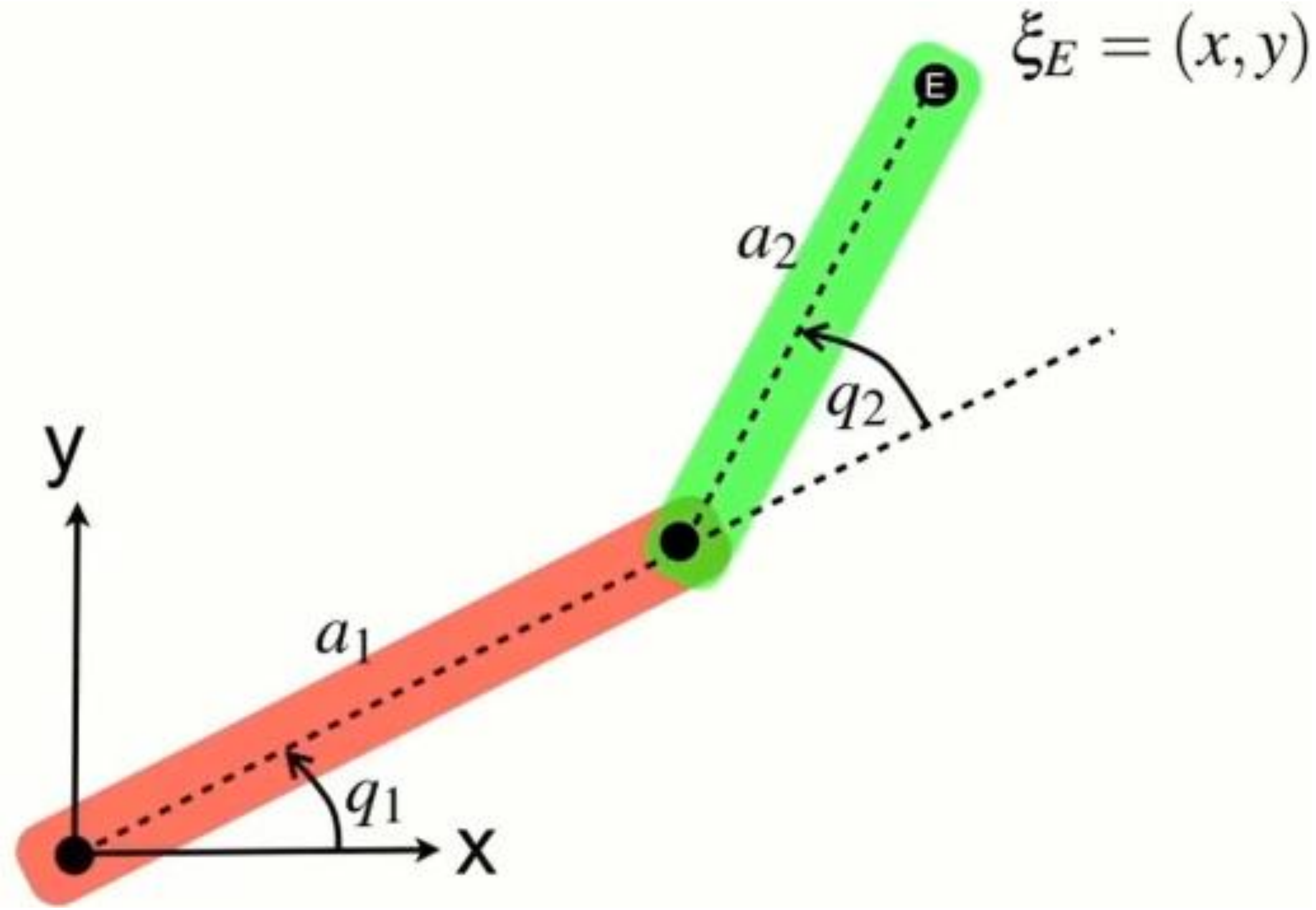
- Given pose of the end-effector ξ_E what are the required joint coordinates?

$$q = \mathcal{K}^{-1}(\xi_E)$$

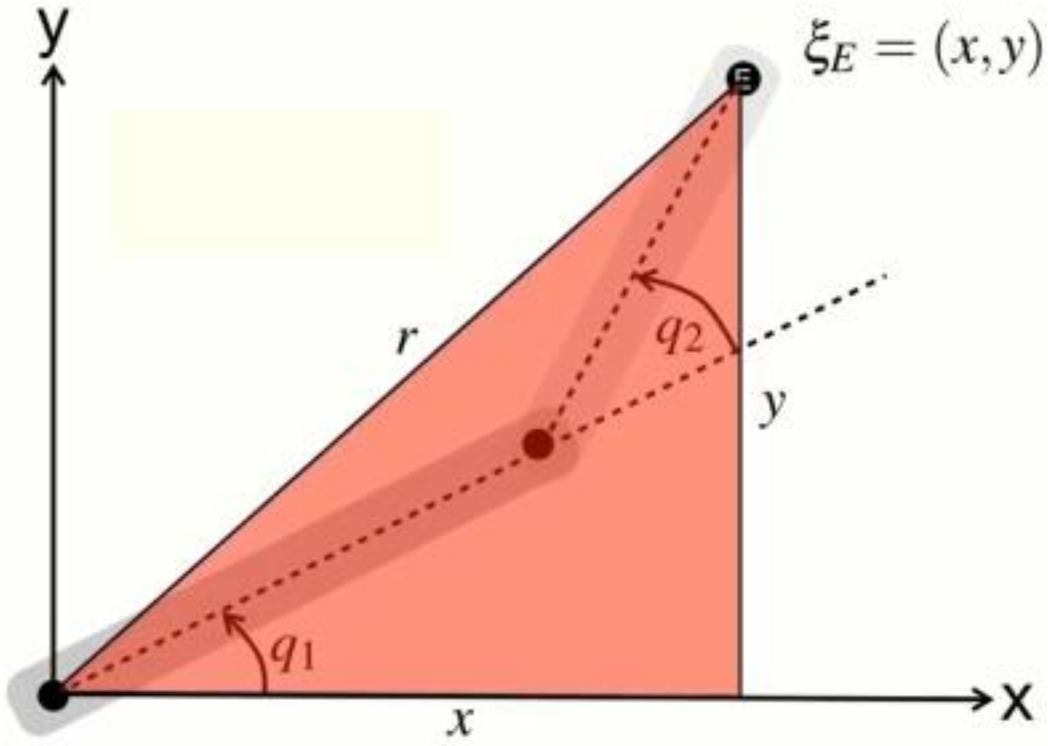
- Inverse kinematics does not have a unique solution, a particular end-effector pose can be achieved by more than one joint configuration.
- Two approaches can be used to determine the inverse kinematics :

    1. Closed-form or analytic solution
    2. Iterative numerical solution

Inverse Kinematics

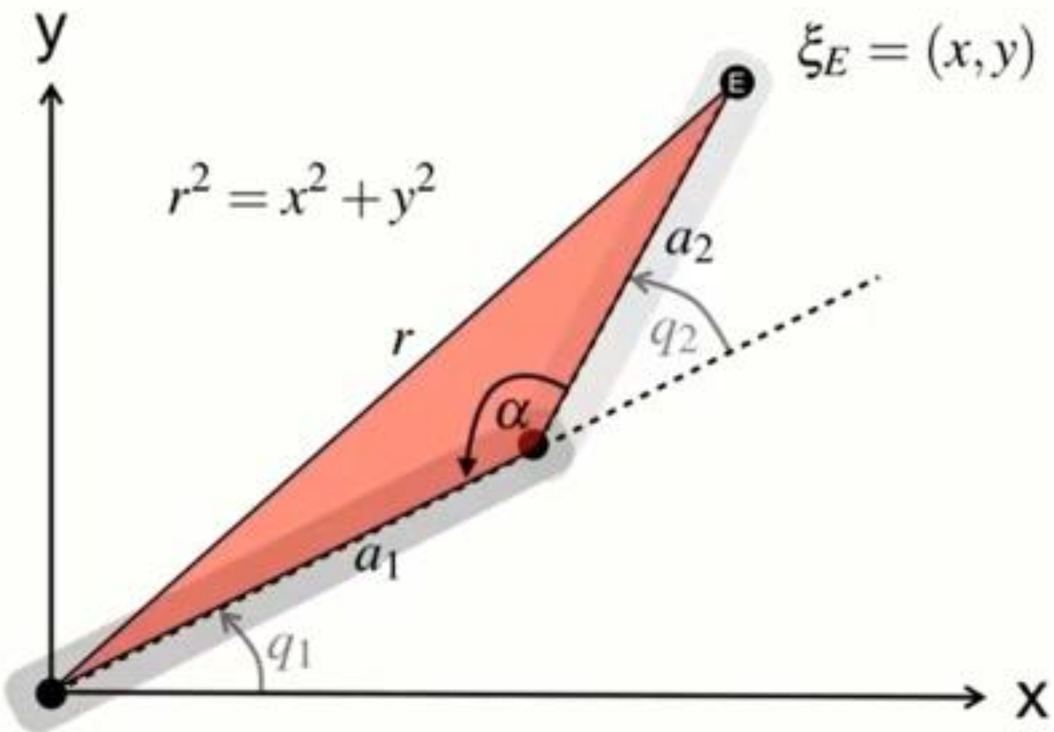


Finding the joint angles



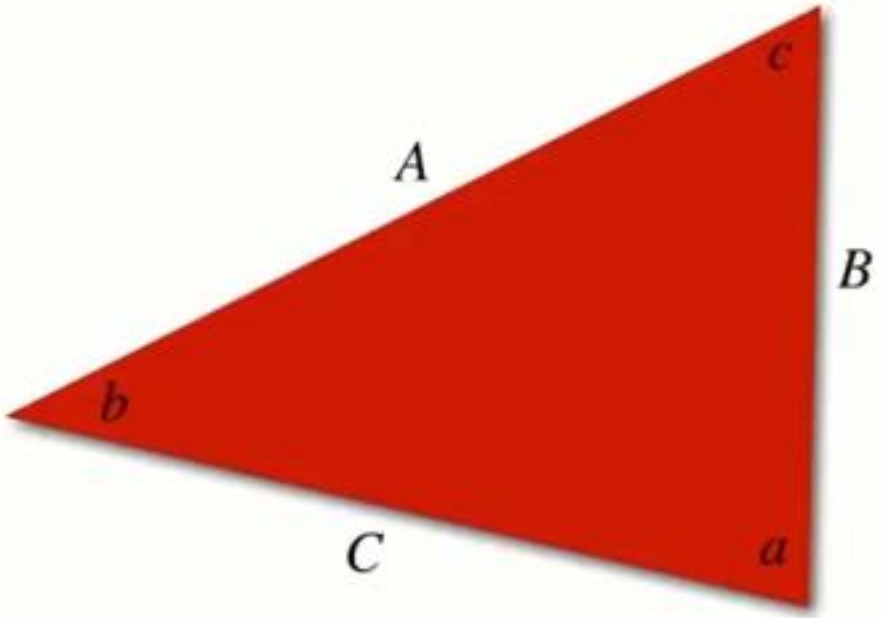
- Apply Pythagoras

Finding the joint angles

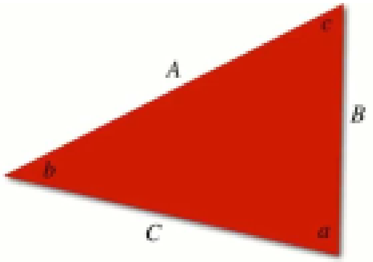


- Apply the cosine rule

Recap: the cosine rule



Sides: A, B, C
Angles: a, b, c

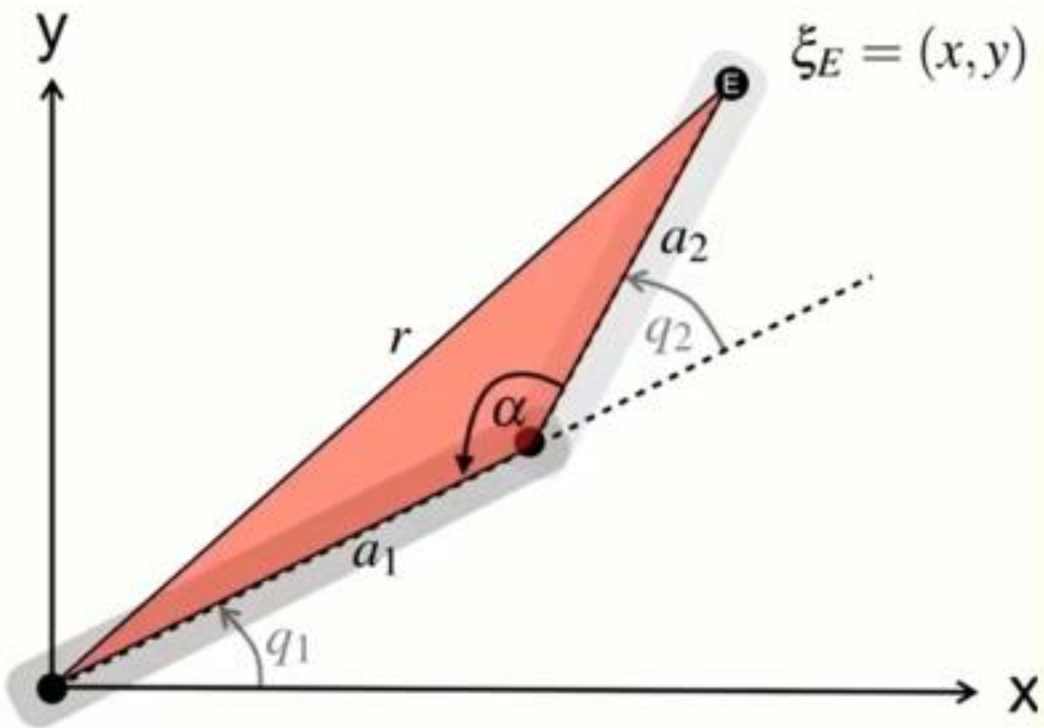


$$A^2 = B^2 + C^2 - 2BC\cos a$$

Finding the joint angles

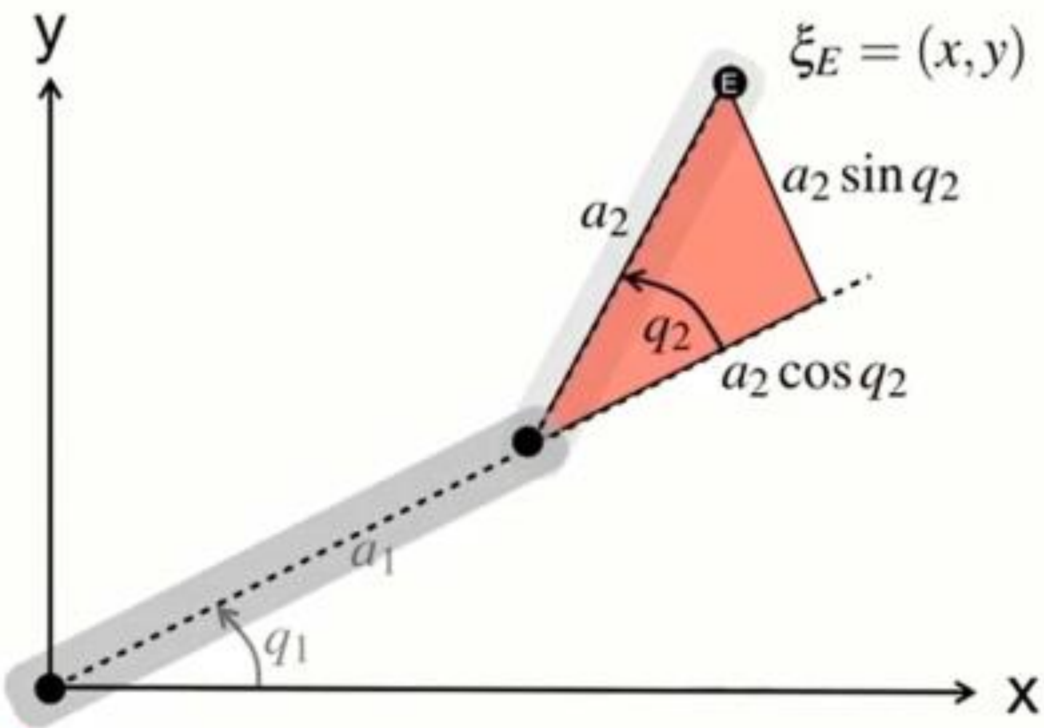
$$r^2 = x^2 + y^2$$

$$\xi_E = (x, y)$$



- Apply the cosine rule

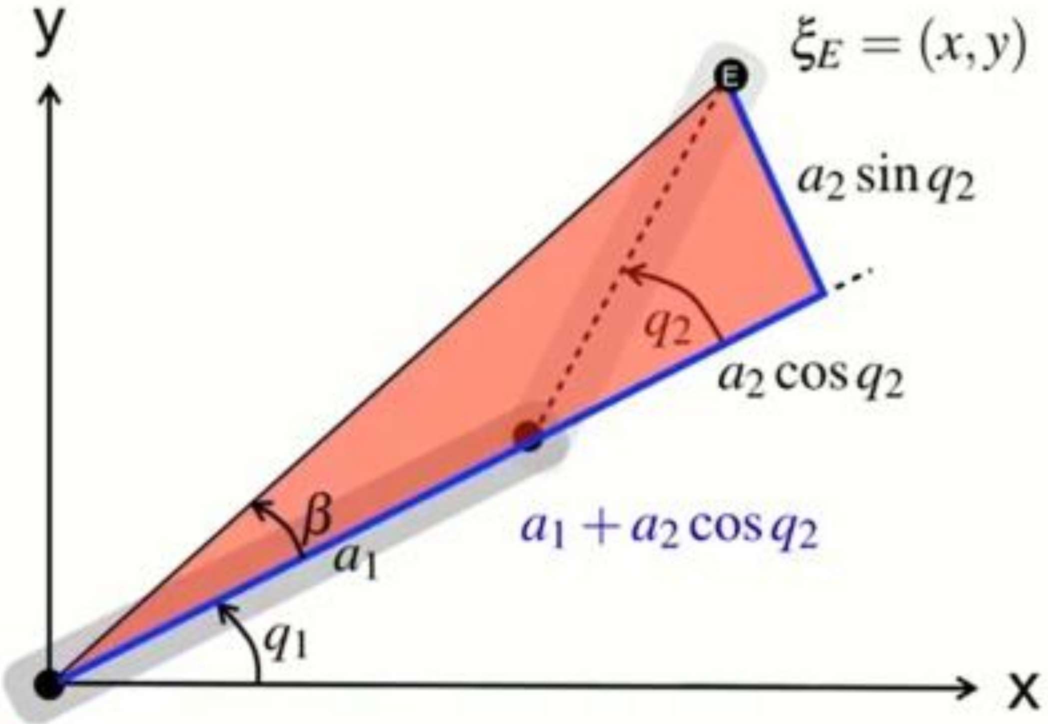
Finding the joint angles



$$\sin q_2 = \sqrt{1 - \cos q_2^2}$$

- Apply simple trigonometry

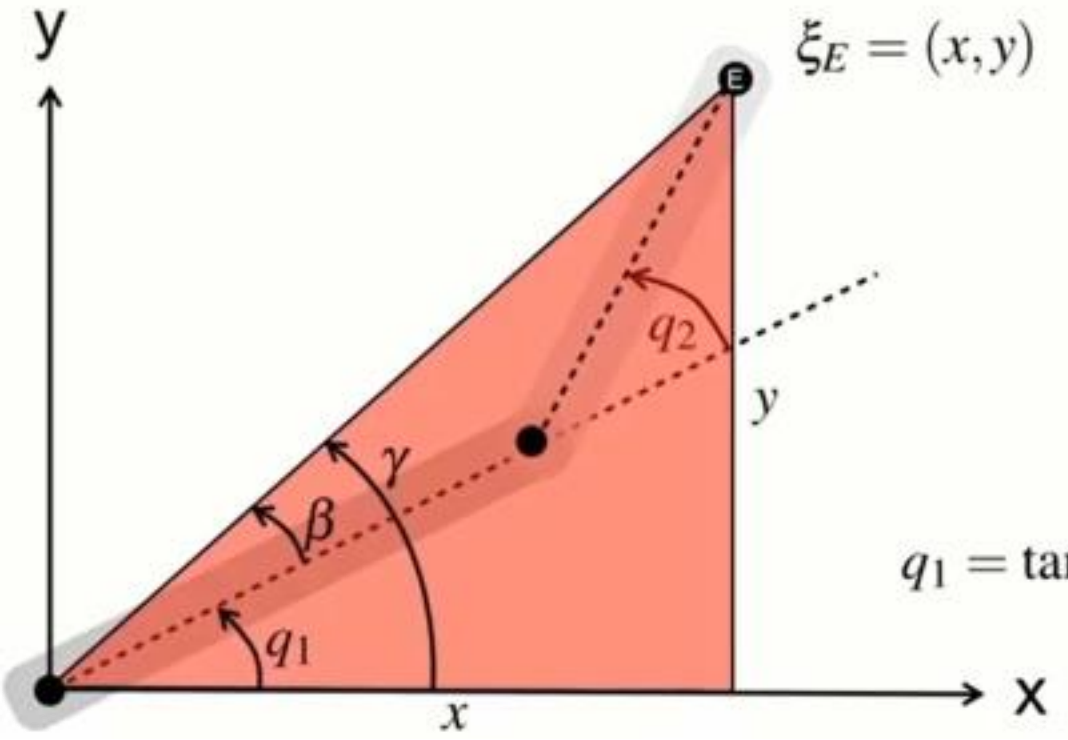
Finding the joint angles



$$\beta = \tan^{-1} \frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}$$

- Apply simple trigonometry

Finding the joint angles



$$\xi_E = (x, y)$$

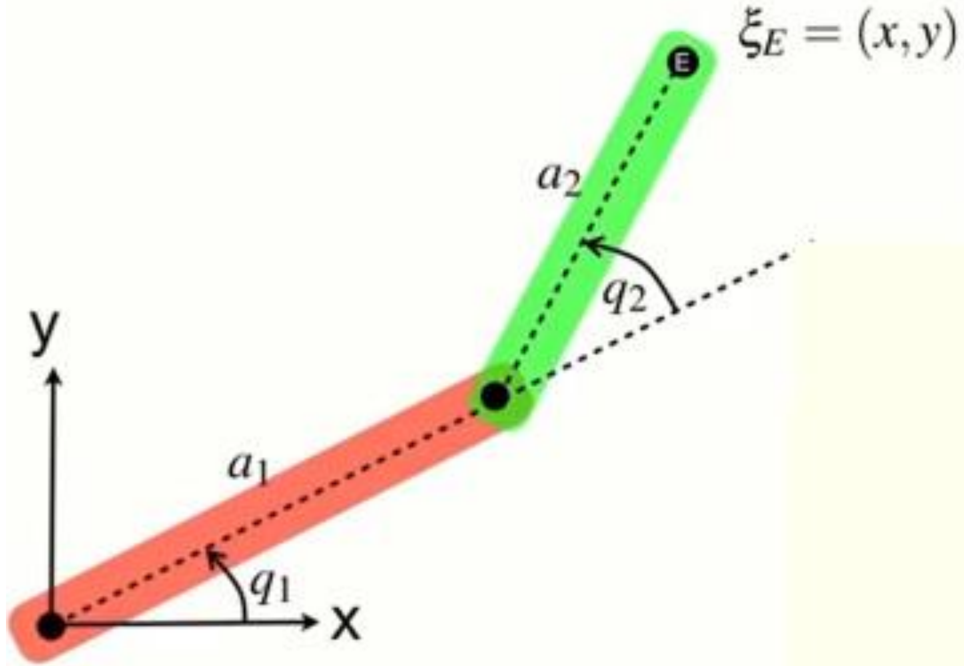
$$\beta = \tan^{-1} \frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}$$

$$\gamma = \tan^{-1} \frac{y}{x}$$

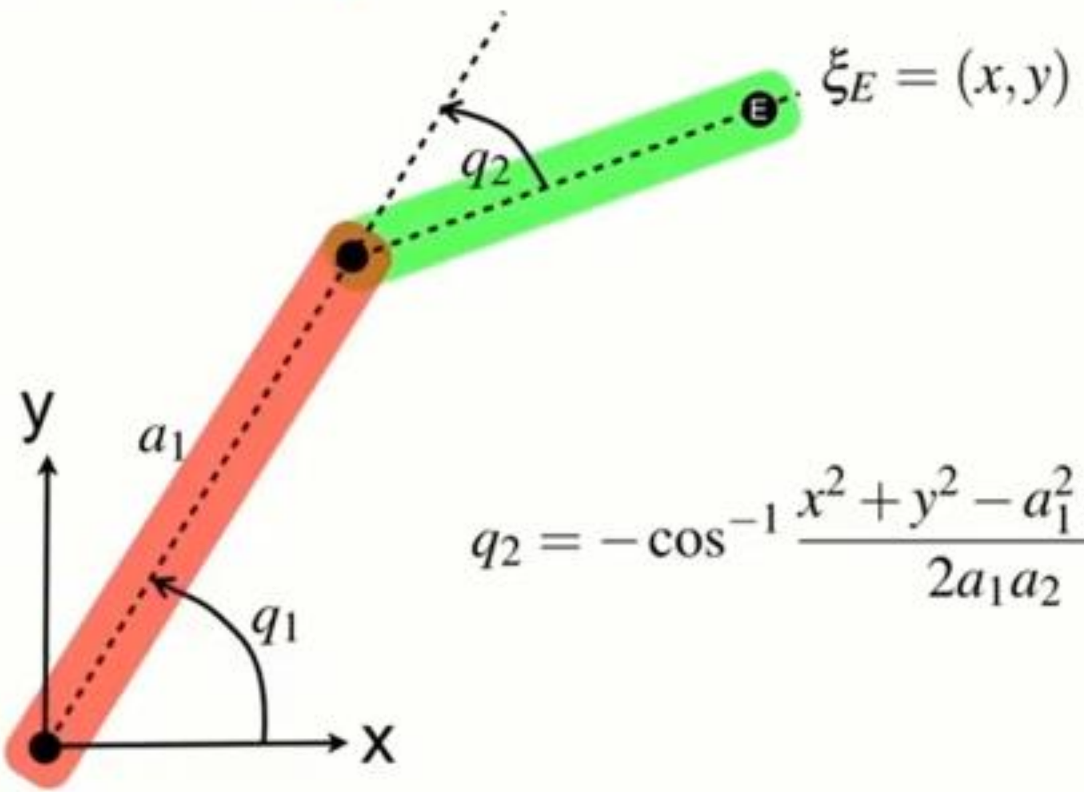
$$q_1 = \gamma - \beta$$

$$q_1 = \tan^{-1} \frac{y}{x} - \tan^{-1} \frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}$$

Tool tip pose

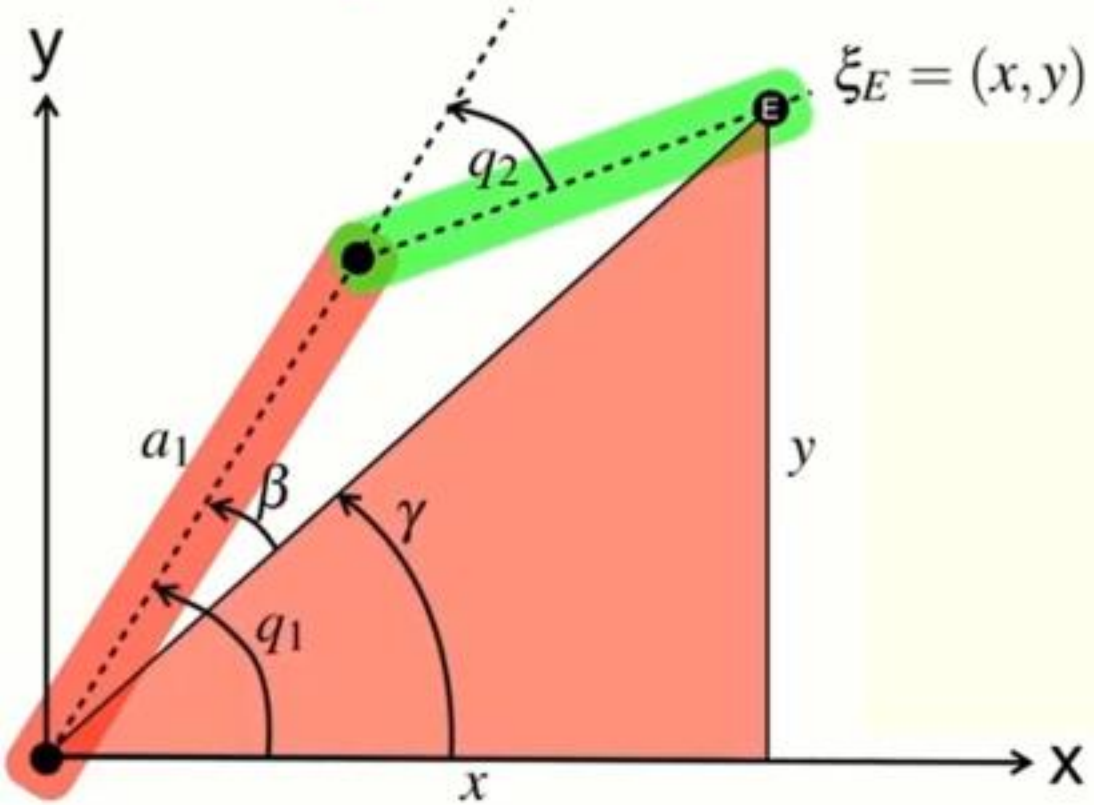


Tool tip pose

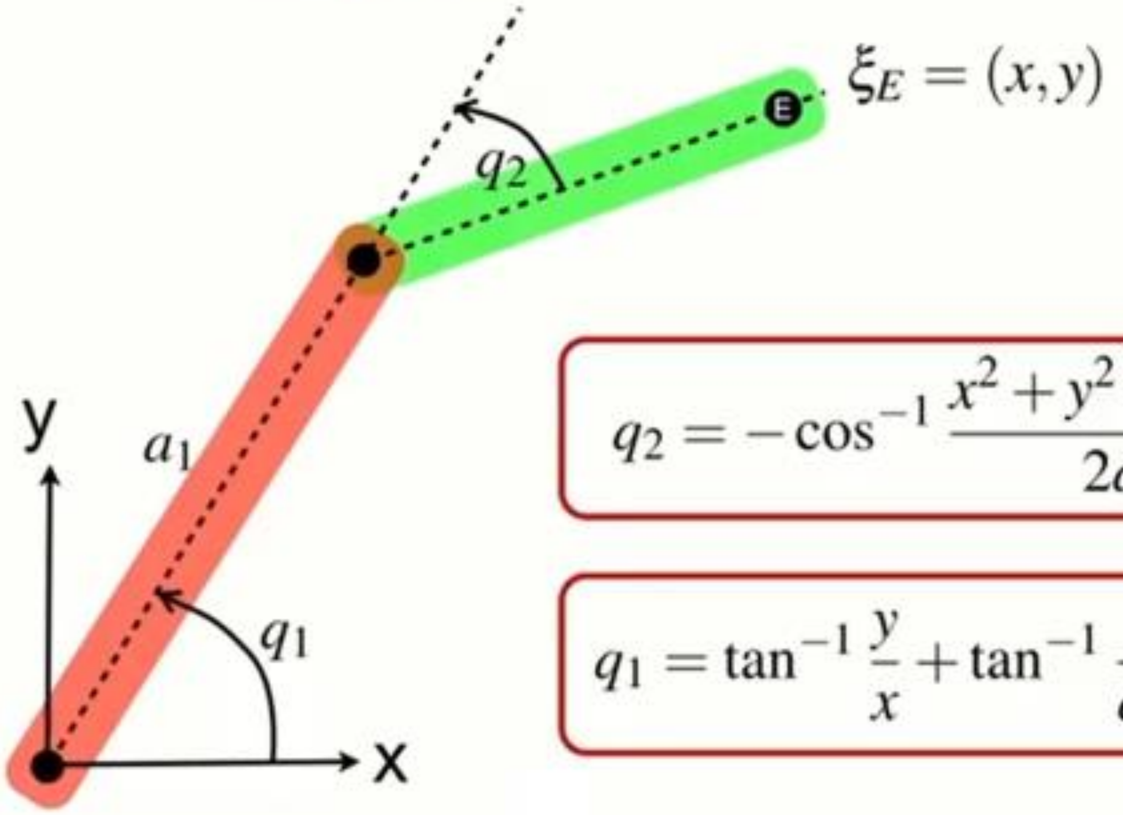


$$q_2 = -\cos^{-1} \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

Tool tip **pose**



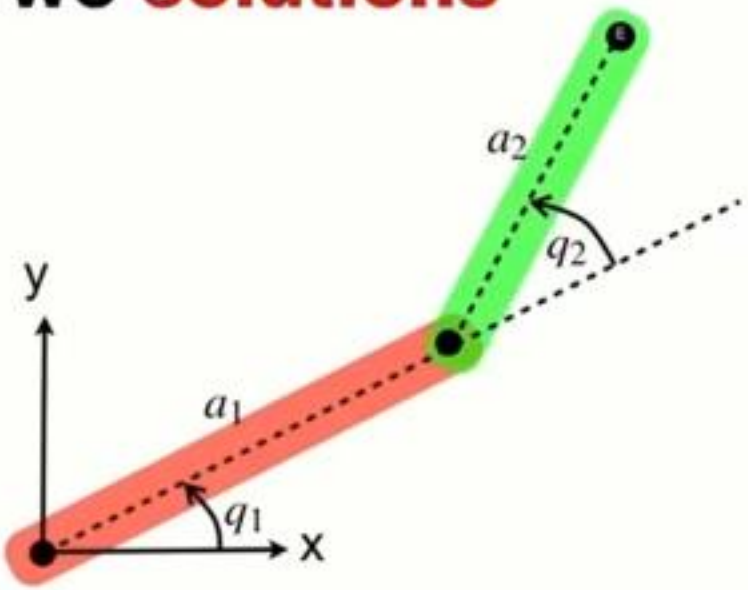
Tool tip **pose**

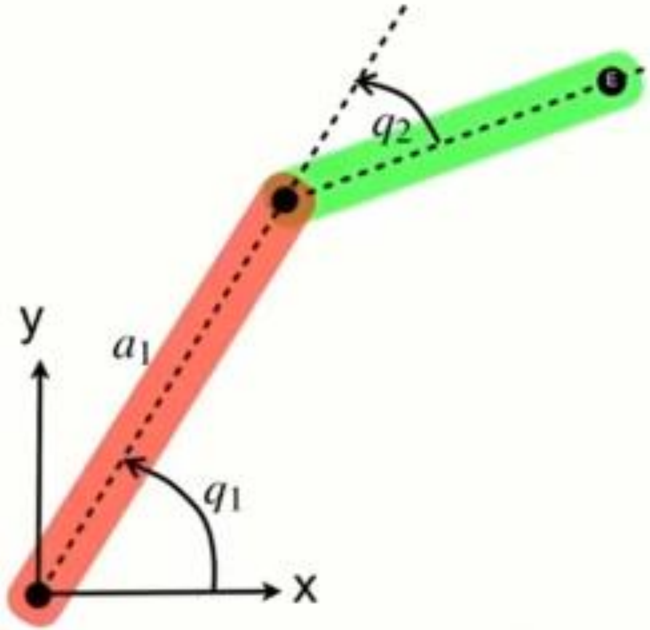


$$q_2 = -\cos^{-1}\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}$$

$$q_1 = \tan^{-1}\frac{y}{x} + \tan^{-1}\frac{a_2\sin q_2}{a_1 + a_2\cos q_2}$$

Two solutions





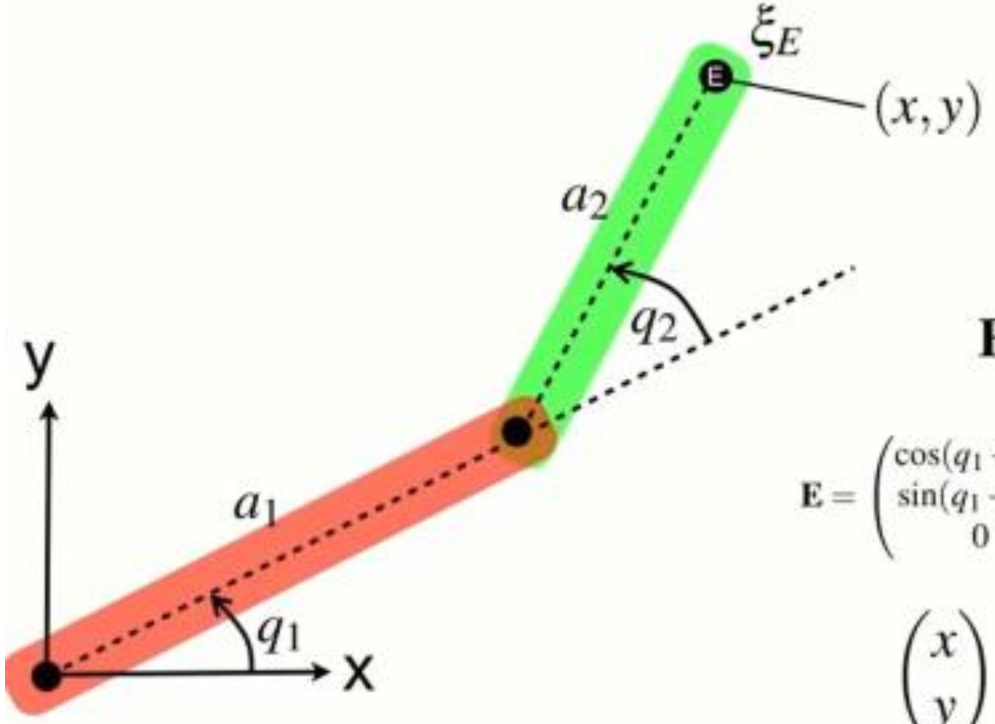
$$q_2 = \cos^{-1}\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}$$

$$q_1 = \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{a_2\sin q_2}{a_1+a_2\cos q_2}$$

$$q_2 = -\cos^{-1}\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}$$

$$q_1 = \tan^{-1}\frac{y}{x} + \tan^{-1}\frac{a_2\sin q_2}{a_1+a_2\cos q_2}$$

Tool tip pose

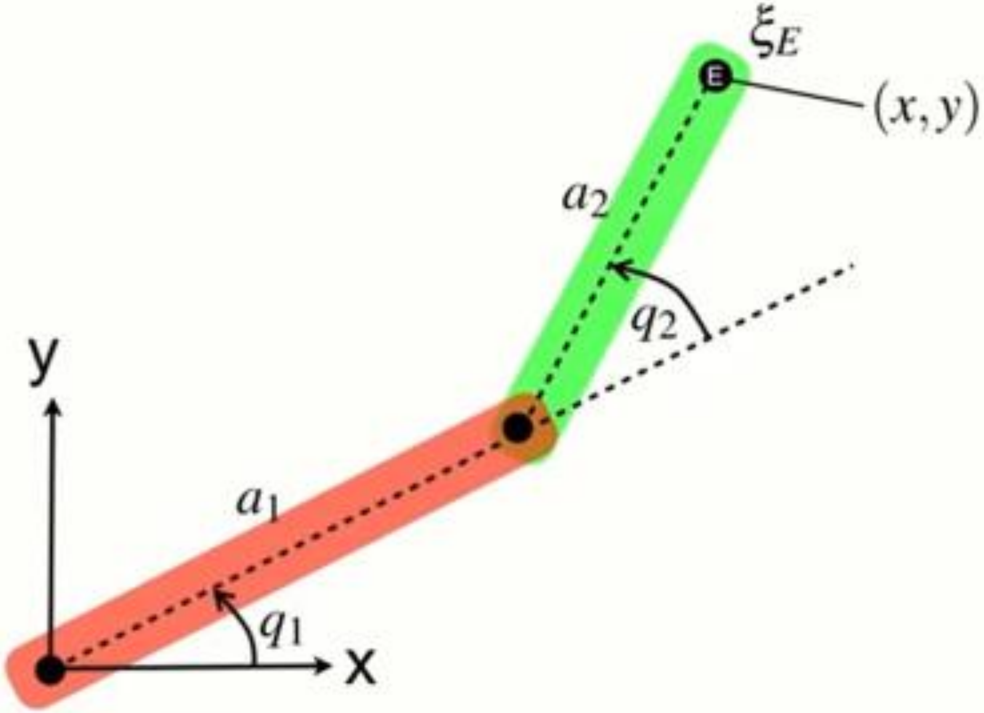


$$\mathbf{E} = \mathbf{R}(q_1)\,\mathbf{T}_x(a_1)\,\mathbf{R}(q_2)\,\mathbf{T}_x(a_2)$$

$$\mathbf{E} = \begin{pmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & a_2\cos(q_1+q_2)+a_1\cos q_1 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & a_2\sin(q_1+q_2)+a_1\sin q_1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_2\cos(q_1+q_2)+a_1\cos q_1 \\ a_2\sin(q_1+q_2)+a_1\sin q_1 \end{pmatrix}$$

Finding the joint angles



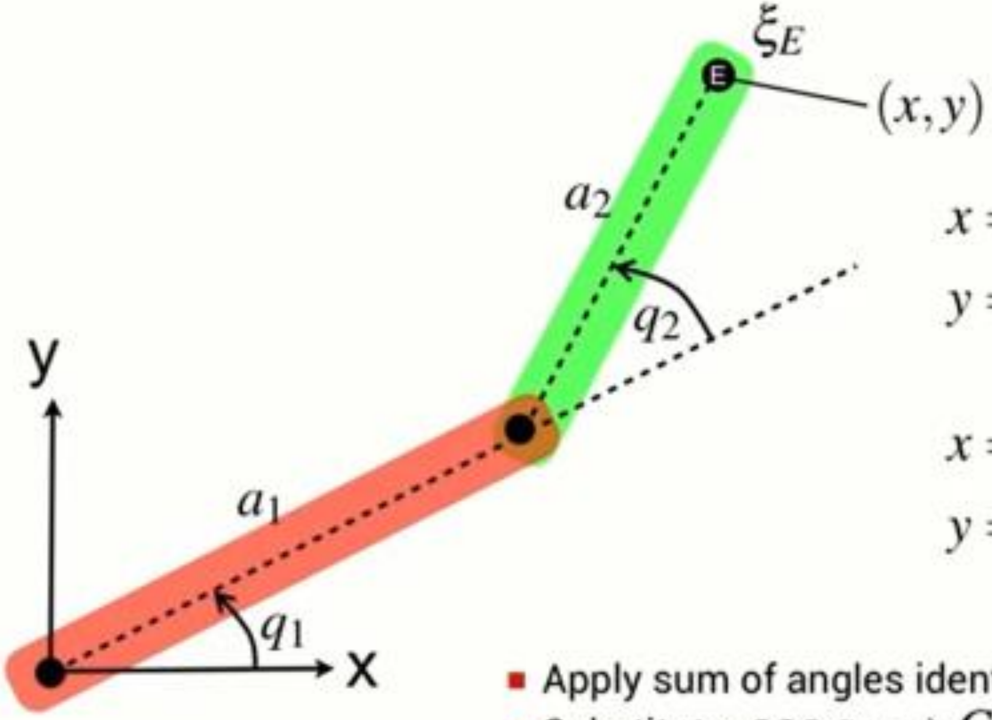
$$x = a_2 \cos(q_1 + q_2) + a_1 \cos q_1$$
$$y = a_2 \sin(q_1 + q_2) + a_1 \sin q_1$$

- Square and add

$$x^2 + y^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos q_2$$

$$\cos q_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

Finding the joint angles

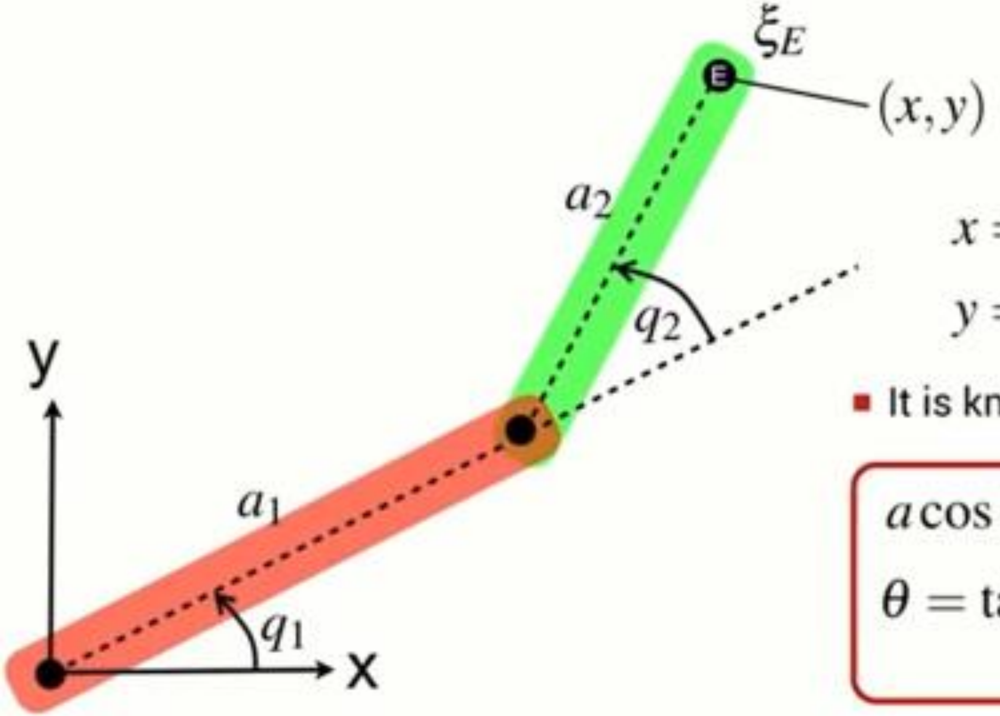


$$x = a_2 \cos(q_1 + q_2) + a_1 \cos q_1$$
$$y = a_2 \sin(q_1 + q_2) + a_1 \sin q_1$$

$$x = (a_1 + a_2 C_2) \cos q_1 - a_2 S_2 \sin q_1$$
$$y = (a_1 + a_2 C_2) \sin q_1 + a_2 S_2 \cos q_1$$

- Apply sum of angles identities
- Substitute $\cos q_2 \to C_2, \sin q_2 \to S_2$

Finding the joint angles

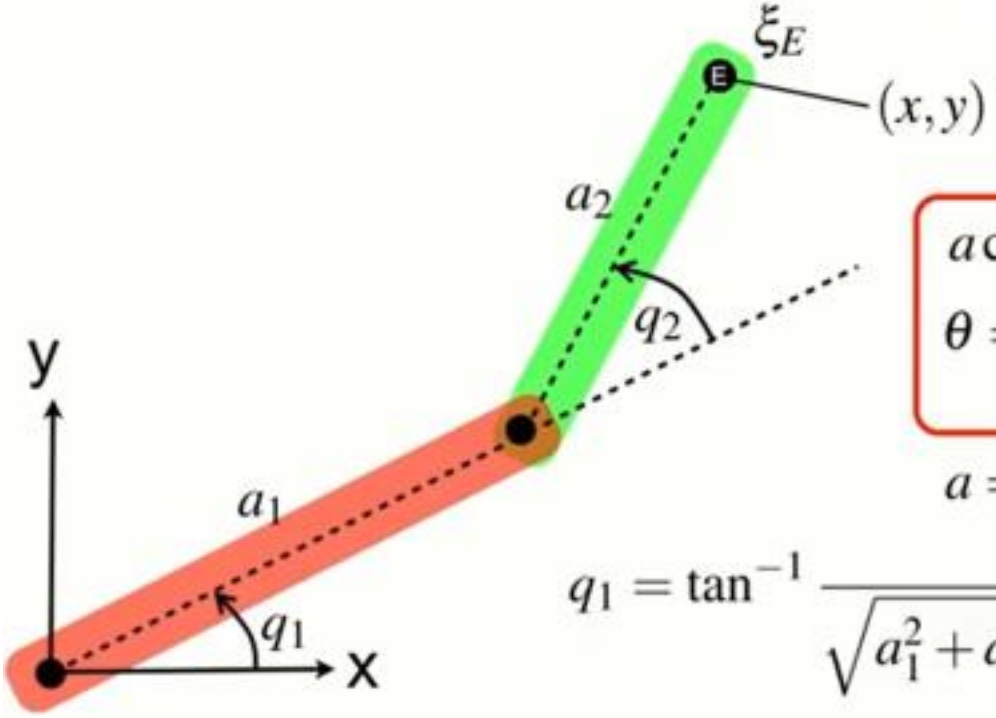


$$x = (a_1 + a_2 C_2)\cos q_1 - a_2 S_2 \sin q_1$$
$$y = (a_1 + a_2 C_2)\sin q_1 + a_2 S_2 \cos q_1$$

- It is known that

$$a\cos\theta + b\sin\theta = c$$
$$\theta = \tan^{-1}\frac{c}{\pm\sqrt{a^2 + b^2 - c^2}} - \tan^{-1}\frac{a}{b}$$

Finding the joint angles



$$x = (a_1 + a_2 C_2)\cos q_1 - a_2 S_2 \sin q_1$$

$$y = (a_1 + a_2 C_2)\sin q_1 + a_2 S_2 \cos q_1$$

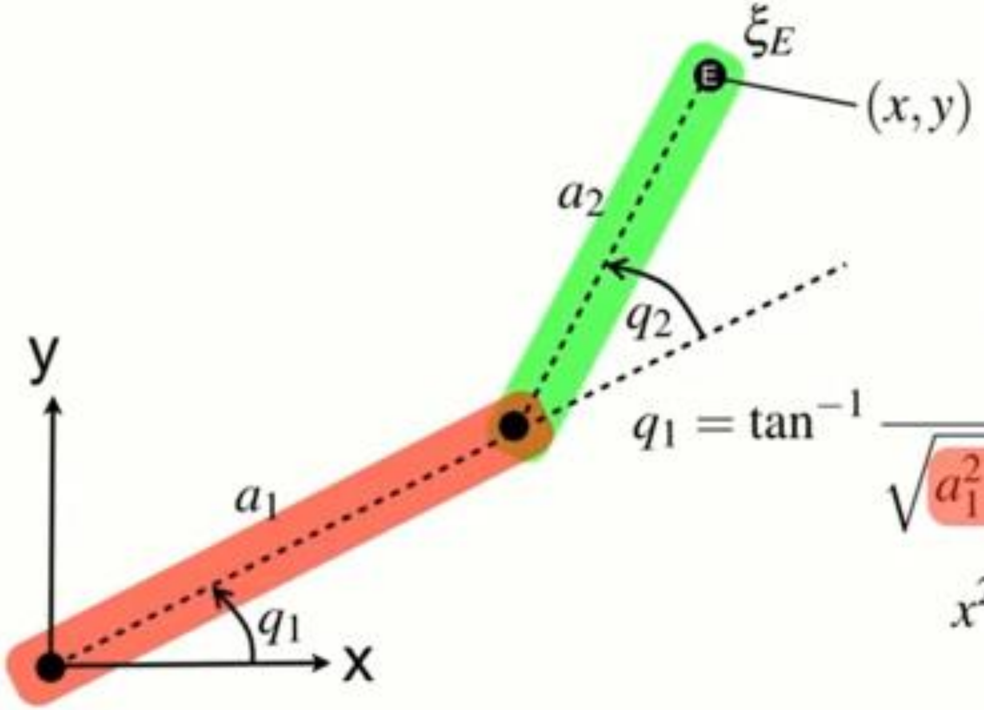
$$a\cos\theta + b\sin\theta = c$$

$$\theta = \tan^{-1}\frac{c}{\pm\sqrt{a^2+b^2-c^2}} - \tan^{-1}\frac{a}{b}$$

$$a = a_2 S_2, b = a_1 + a_2 C_2, c = y$$

$$q_1 = \tan^{-1}\frac{y}{\sqrt{a_1^2 + a_2^2 + 2a_1 a_2 C_2 - y^2}} - \tan^{-1}\frac{a_2 S_2}{a_1 + a_2 C_2}$$

Finding the **joint angles**



$$q_1 = \tan^{-1} \frac{y}{\sqrt{a_1^2 + a_2^2 + 2a_1a_2C_2 - y^2}} - \tan^{-1} \frac{a_2 S_2}{a_1 + a_2 C_2}$$

$$x^2 + y^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos q_2$$

2-Dimensional (Planar) Robotic Arms

- Closed-form or analytic solution

```
import sympy
a1, a2 = sympy.symbols("a1 a2")
e = ET2.R() * ET2.tx(a1) * ET2.R() * ET2.tx(a2);
q0, q1 = sympy.symbols("q0 q1")
TE = e.fkine([q0, q1])
x_fk, y_fk = TE.t;
x, y = sympy.symbols("x y")
eq1 = (x_fk**2 + y_fk**2 - x**2 - y**2).trigsimp()
q1_sol = sympy.solve(eq1, q1)
eq0 = tuple(map(sympy.expand_trig, [x_fk - x, y_fk - y]))
q0_sol = sympy.solve(eq0, [sympy.sin(q0), sympy.cos(q0)]);
```

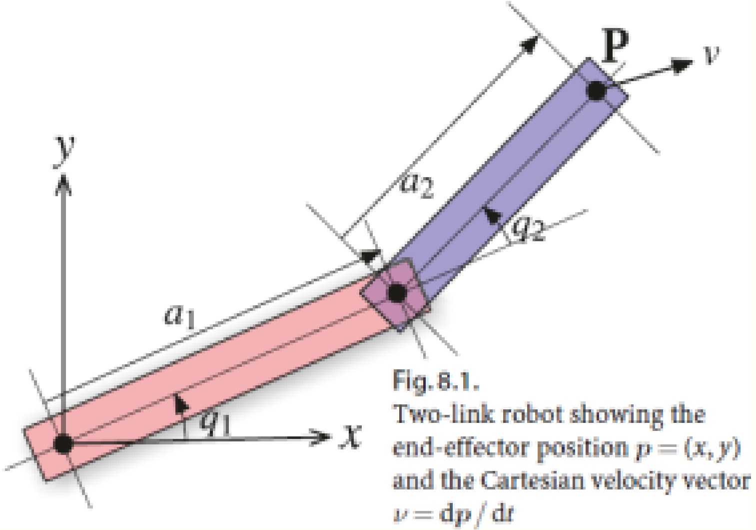


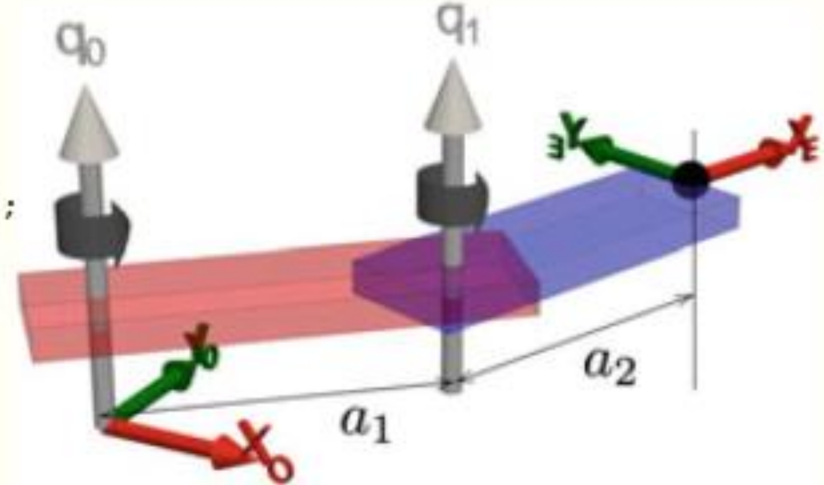
Fig. 8.1. Two-link robot showing the end-effector position $p = (x, y)$ and the Cartesian velocity vector $\nu = dp / dt$

2-Dimensional (Planar) Robotic Arms

- Iterative numerical solution : adjust the joint coordinates until the forward kinematics matches the desired pose.
- An optimization problem : minimize the error between the *forward kinematic* solution and the desired end-effector pose ξ_E^* $\quad \boldsymbol{q}^* = \arg\min_{\boldsymbol{q}} \| \mathcal{K}(\boldsymbol{q}) \ominus \boldsymbol{\xi}_E^* \|$
- Define an error function based on the end-effector position error, not its orientation.

$$E(q) = \| [\mathcal{K}(\boldsymbol{q})]_t - (x^*, y^*)^\top \|$$

```
e = ET2.R() * ET2.tx(1) * ET2.R() * ET2.tx(1);
pstar = np.array([0.6, 0.7]); # desired position
E = lambda q: np.linalg.norm(e.fkine(q).t - pstar);
sol = optimize.minimize(E, [0, 0]);
sol.x
e.fkine(sol.x).printline()
```



3-Dimensional Robotic Arms

- Closed-form inverse kinematics using the Denavit-Hartenberg model for the Puma robot.
- Puma 560 is a 6-axis robot arm with a spherical wrist we use the method *ikine6s* to compute the inverse kinematics using a closed-form solution

```
mdl_puma560
qn
T = p560.fkine(qn)
qi = p560.ikine6s(T) % why qi not equal to qn
p560.fkine(qi)
```